GENERALIZED AUTOREGRESSIVE HETEROSCEDASTIC [GARCH (p, q)] MODELS ON CRUDE OIL PRICES IN NIGERIA

BY

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Abstract

The paper studied the volatility GARCH (p,q), GARCH(1,1), GARCH (1,2) GARCH(2,1) and GARCH(2,2) models using the data from Central Bank of Nigeria from 2015 to 2020. Akaike information criterion (AIC) and Schwarzt Information criterion was used to determine the optimal forecasting model for the crude oil price in Nigeria. This research aimed at characterizing a good volatility model to forecast and capture the commonly held stylized facts about conditional volatility. These include the persistence in volatility, mean reverting behavior, asymmetric impact of negative versus positive return innovations and the possibility that exogenous or pre-determined variables may have a significant influence on volatility. The GARCH (1,1) model is able to model and forecast better than other competing models. The forecast results show a slight upward movement in the crude oil prices.

Keywords: *GARCH (p,q), Volatility, Stylized facts.*

1.0 Introduction

As the structure of world industries changed in 1970s, the expectation of the oil market has continually grown to have now become the world's biggest commodity market. This market has developed from primarily physical product activity into sophisticated financial market. Over the last decade, crude oil markets have matured greatly, and their range and depth could allow a wide range of participants such as crude oil producers, crude oil physical traders, and refining and oil companies, to hedge oil price risk. Risk in the crude oil commodity market is likely to occur due to unexpected jumps in global oil demand, a decrease in the capacity of crude oil production and refinery capacity, petroleum reserve policy, OPEC spare capacity and policy, major regional and global economic crises, and geopolitical risks Tansuchant, Chang and Mcleer (2010). A development in the global economy posing a great challenge to policy makers across countries is the increasing spate of fluctuations in oil prices.

The transmission mechanisms through which oil prices have impact on real economic activity include both supply and demand channels. The supply side effects are related to the fact that crude oil is a basic input to production, and consequently an increase in oil price leads to a rise in production costs that induces firms to lower output. Oil prices changes also entail demand-side effects on consumption and investment. Consumption is affected indirectly through its positive relation with disposable income.

Over the last four decades, oil has been the main driver of Nigeria's economy. Nigeria's oil sector accounts for over 90% of total export earnings and over 30% of the country's GDP. The discovery of oil and the exploitation and export of same in commercial quantities which began in the early 1970s led to the neglect of virtually other sectors of the economy particularly agriculture and manufacturing, turning the economy into a near mono-product economy. The precarious dependence of Nigeria's economy on the crude oil sector has tended to retard the growth of the economy as the price of crude oil in the international crude oil market is highly volatile.

It has always set budget that either overshoots or undershoots the level that is consistent with the expected revenue for that fiscal year; thereby making the economy to incur debt in order to fully implement the budget in the event of an overshoot of the budget for that fiscal year or under-utilized the revenue in the event of an undershoot of the budget for that fiscal year. Since the economy depends on revenue from crude oil and the international price of crude oil is exogenously determined, an accurate forecast of this crude oil price will be of upmost significance for the Nigerian government to be able to situate and benchmark its budget and utilize available resources optimally.

There are different ways for modeling changes of prices over time. A commonly used model is the autoregressive conditionally heteroscedastic (ARCH) model introduced by Engle (1982) in which the conditional variance is a function of the squared past values of the series including time $t-1$. Consequently, the volatility is observable at time t-1. This model has been extended in different directions. The most popular of them is the generalized autoregressive heteroscedastic (GARCH) model which was proposed by Bollerslev, after four years of introduction of ARCH models and it lets conditional variance depend on the squared past observations and previous variances.

1.1 Aim and Objectives of the Study

The purpose of this study is to characterize a good volatility model by its ability to forecast and capture the commonly held stylized facts about conditional volatility. The stylized facts include such things as the persistence in volatility, its mean reverting behavior. Using Volatility Models GARCH (p,q), to analyze crude oil price data based on Nigeria with a view to achieve the following objectives:

i. To determine the optimal forecasting model for the crude oil price data in Nigeria.
ii. To determine the accuracy of the models.

2.0 Review of Related Literature

Research has been conducted on the volatility of crude oil spot, forward and futures return. Lanza et al. (2006) applied the constant conditional correlation (CCC) model of Bollerslev (1990) and the dynamic conditional correlation (DCC) model of Engle (2002) for West Texas Intermediate (WTI) oil forward and futures returns. Manera et al. (2006) used CCC, the vector autoregressive moving average (VARMA – GARCH) model of Ling and McAleer (2002), the VARMA – Asymmetric GARCH model of McAleer et al. (2009), and DCC to spot and forward return in the Tapis Market. Recently, Chang et al. (2009a, 2009b, 2009c) estimated multivariate conditional volatility and examined volatility spillovers for the returns of spot, forward and futures returns for Brent, WTI, Dubai and Tapis to aid risk diversification in the crude oil markets.

Over the years, several studies have applied GARCH type models to examine volatility in relation to trade, stock markets and exchange rates. Adamu (2005) for example explores the impact of exchange–rate volatility on private investment and confirms an adverse effect. Mordi (2006) employing GARCH model argues that failure to properly manage exchange rates can induce distortions in consumption and production patterns and that excessive currency volatility creates risks with destabilizing effects on the economy.

Lescaroux and Migno (2008) in three panels of OPEC members, other major oil exporting countries and some oil importing countries investigated the links between oil prices and various macroeconomic and financial variables including GDP, CPI, unemployment rate and bond price. Using causality tests, evaluation of cross-correlations between the cyclical components of the series and cointegration analysis, they found various relationships between oil prices and macroeconomic variables in both the short and long run. In long run, specifically, ―the causality generally running from oil prices to the other variables. And, finally, Kireyev (2000), using the mean-group estimator in a PVAR approach, analyzed the effects of both internal and external shocks on macroeconomic movements in 18 Arab countries. In his study, based on the data for last three decades of 20th century, kireyev classified sample countries to various groups and compared the pattern of dynamic adjustments between these groups.

Wilson, David, Inyiama and Beatrice (2012) examined the relationship between oil price volatility and economic development in Nigeria. Applying Ordinary Least Square and Granger Causality Test, the study shows that there is no significant relationship between oil price volatility and key macroeconomic variables (Real GDP, inflation, interest rate and exchange rate).

Contrarily, the study of oil price shocks and volatility of selected macroeconomic indicators in Nigeria carried out by Taiwo, Abayomi and Damilare (2012) using Johasen Cointegration Test and Error Correction Model indicated that crude oil price, stock price and exchange rate have significant influence on the growth of the Nigerian economy. Oriakhi and Osaze (2013) examined the consequences of oil price volatility on the growth of the Nigeria economy within the period 1970 to 2010. With the use of VAR model, the study find that oil price volatility has direct impact on government expenditure, real exchange rate, and real import while real GDP and inflation are indirectly influenced by the oil price volatility. By implication the study shows that changes in oil price determine government expenditure which in turn determines the growth of the Nigerian economy.

3.0 Data and Methodology

The study is focused on monthly crude oil prices in (American Dollar) Nigeria, for the period of January 2015 to July 2020. The data are obtained from Central Bank of Nigeria website (www.cbn.gov.ng).

We apply GARCH(p,q) type models ie GARCH (1, 1) and GARCH(1, 2) and GARCH (2,1) and GARCH (2,2). We evaluate the best model using two information criteria: Akaike criterion (AIC) and Scwarzt information criterion (SIC)

3.1 GARCH (p,q) Model and its Properties

In practice, it is often found that large number of lag p, and large number of parameters, are required to obtain a good model fit of ARCH (p) model. Bollerslev in 1986 proposed Generalized ARCH or GARCH (p, q) model to solve this problem with the following formulation:

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + --- + \alpha_p \varepsilon_{t-p}^2 + \beta_1 h_{t-1} + --- + \beta_q h_{t-q}$$

Where,

h_t is the volatility at day t – i

$\alpha_0 > 0$

$\alpha_i \geq 0$ for i = 1, ----, p

$\beta_i \geq 0$ for i = 1, ---, q

ε_{t-1}^2 *and* h_t are as previously defined.

Under GARCH (p,q) model, the conditional variance of $\varepsilon_t, h_{t,}$ depends on the squared innovations in the previous p periods, and the conditional variance in the previous q periods. The GARCH models are adequate to obtain a good volatility model fit for financial time series.

Rearranging the GARCH (p,q) model by defining $\mu_t \equiv \varepsilon_t^2 - h_t$, it follows that

$$\varepsilon_t^2 = \alpha_0 + (\alpha(L) + \beta(L)\varepsilon_t^2 - \beta(L)\mu_t + \mu_t)$$

Where, L is the backshift operator and

$\alpha(L) = \alpha_1 L + \cdots + \alpha_p L^p$

$\beta(L) = \beta_1 L + \cdots + \beta_q L^q$

Which is an ARMA (max (q,p), q) model for ε_t^2. By standard argument, the model is covariance stationary if and if all the roots of $(1 - \alpha(L) - \beta(L))$ lie outside the unit circle. The ARMA representation in 3.30 allows for the use of time series techniques in the identification of the order of p and q. for the sake of simplicity however, we are going to examine the GARCH (1,1) model and investigate all the features of stylized facts exhibited by the model.

The standard GARCH (1, 1) model process is specified as:

$h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1} - - - 3.25$

Where,

β_1 measures the extent to which a volatility shock today feeds through into the next period's volatility.

$(\alpha_1 + \beta_1)$ measures the rate at which this effect dies over time

h_{t-1} is the volatility at day *t -1*.

The conditional variance equation of GARCH (1,1) model contains a constant term, news about volatility from the previous period, measured as the lag of previous term squared residuals.

3.2 Forecasting in GARCH (1,1) Model

Using GARCH (1,1) model it is easy to construct multi period forecasts of volatility. When $\alpha_1 + \beta_1 < 1$, the unconditional variance of ε_t is $\alpha_0/(1 - \alpha_1 - \beta_1)$, which is also known as the long run variance of ε_t. If we re – write the following GARCH (1,1) as

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$
$$= \alpha_0 + \alpha_1 \left(\varepsilon_{t-1}^2 - h_{t-1} \right) + \left(\alpha_1 + \beta_1 \right) h_{t-1}$$

The multi period volatility forecast reverts to its unconditional mean at the rate of $\left(\alpha_1 + \beta_1 \right)$.

3.3 Test for Unit Root

In order to make inferences on time series, they must be stationary. However, most of the financial time series do not satisfy the requirements of stationarity so that they have to be converted to stationary processes before modeling. Many test statistics have been developed to check whether the series contains unit roots or not. The most popular of them is Dickey-Fuller test. Dickey and Fuller (1979) introduced Dickey – Fuller (DF) test statistic to test whether the series contains unit root or not. However, it can be any other processes also. Because of this Augmented Dickey–Fuller (ADF) test statistics and KPSS Test (Kwitkoswski, Philips, Schmidt and Shin) for unit root has developed in the same manner to check the stationarity of the series.

3.3.1 Augmented Dickey Fuller Test for Unit Root

The regression model for the test is given as:

$$\Delta y_t = \alpha y_{t-1} + x_t^1 \delta + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + --- + \beta_p \Delta y_{t-p}$$

The hypothesis Testing:

$H_0 : \alpha = 0$ (the series contains unit root(s))

$H_1 : \alpha < 0,$ (the series is stationery)

Test statistic: $t_\alpha = \hat{\alpha} / se\left(\hat{\alpha} \right)$

Reject H_0 if t_α is less than asymptotic critical values.

Where,

Δy_t = the differenced series

y_{t-1} = the intermediate previous observation

$\beta_1 ---- \beta_2$ = the coefficients of the lagged difference term up to lag p

x_t = the optional exogenous regressor which may be constant, or a constant trend.

$\alpha \& \delta$ = parameter to be estimated

3.3.2 KPSS Test (Kwiatkowski, Philips, Schmidt and Shin) for unit root

The integration properties of series y_t may also be investigated by testing:

$H_0: y_t \sim I(0)$ Vs $H_A: y_t \sim I(1)$

That is, the null hypothesis that the data generating process (DGP) is stationery is tested against a unit root. Kwiatkowski, Philips, Schmidt, and Shin (1992) have derived a test for this pair of hypotheses. If there is no linear trend term, they start from DGP.

$y_t = x_t + Z_t$ where, x_t is a random walk i.e. $x_t = x_{t-1} + v_{t,} v_t \sim iid(0, \sigma_v^2)$ and z_t is stationery process.

Test statistics: $KPSS = \frac{1}{T^2} \sum_{t=1}^{T} \frac{S_t^2}{\hat{\sigma}_\infty^2}$ where, $S_t = \sum_{j=1}^{t} \hat{w}_j$ with $\hat{w}_j = y_t - \bar{y}$ and $\hat{\sigma}_\infty^2$ is an estimator of $\sigma_\infty^2 = \lim_{T \to \infty} T^{-1} \operatorname{var}\left(\sum_{t=1}^{T} Z_t \right)$.

That is $\hat{\sigma}_\infty^2$ an estimator of the long – run variance of the process Z_t

$w_j = 1 - \frac{j}{l_q + 1}$ is a Bartlett window with truncated lag l_q.

Reject null hypothesis if the test statistic is greater than the asymptotic critical values.

3.4 Testing for Auto Regressive Conditional Heteroscedasticity (Arch) Effects.

3.5.1 ARCH – LM TEST

It is usually good practice to test for presence of ARCH effects in the residuals before estimating a full a GARCH model for a financial time series. This test is used to check if the error ε_t (in ARCH (p) model) is truly a skedastic function.

The regression is given is given thus:

$$\hat{\varepsilon}_t^2 = \alpha_0 + \alpha_1 \hat{\varepsilon}_{t-1}^2 + --- + \alpha_p \hat{\varepsilon}_{t-p}^2 + u_t \quad \text{where,}$$

$\alpha_1, ---, \alpha_p$ are the coefficients of the regression and α_0 is the intercept.

$H_0 : \alpha_1 = \alpha_2 = --- = \alpha_p = 0$ there no ARCH effects in the residuals under the null the LM statistic is distributed asymptotically as $\chi^2(p)$ statistic.

Test Statistic: nR^2

The test is from Engle (1982) who drive a LM statistics for testing the null as nR^2, where n is the number of observation and R^2 is from the regression. Reject the null if the P – value is less than critical Value α.

3.6 Test of Normality

The residuals data of crude oil prices are checked for normality. If the dataset is normally distributed, then a parametric statistic like the normal distribution can be assumed. However, if the residuals data are not normally distributed, a non – parametric statistic will be used. We employed the Jacque – Bera (JB) test statistics for the normality test:

3.6.1 JARQUE – BERA Test

Jarque and Bera (1987) have proposed a test for non – normality based on skewness and kurtosis of a distribution. The test checks the pairs of hypotheses:

$H_0 = E\left(u_t^s\right)^3 = 0 and E\left(u_t^s\right)^4 = 3$ The distribution is symmetry and hence normal

$H_1 : E\left(u_t^s\right)^3 \neq 0 and E\left(u_t^s\right)^4 \neq 3$ The distribution is asymmetry and hence non – normal

Test Statistics: $JB = \frac{T}{6}\left[T^{-1}\sum_{t=1}^{T}\left(\hat{u}_t^s\right)^3\right]^2 + \frac{T}{24}\left[T^{-1}\sum_{t=1}^{T}\left(\hat{u}_t^s\right)^4 - 3\right]^2$

is an asymptotic x^2 (2) distribution if the null hypothesis is correct and T is the number of observations. Reject the null if the P – value is less than critical value α.

3.7 Model Selection

The best model was selected using two information criteria: Akaike information criteria (AIC) and Schwarz information criteria (SIC). AIC and SIC considers the accuracy of the model fit and the number of parameters in the model; rewarding a better fit and penalizing an increased number of parameters in the series data. The optimal model that will be selected is the model with the minimum AIC and SIC values. The GARCH models used in this study were fitted using maximum likelihood method

AIC = T ln (residual sum of squares) + 2n,

SIC = T ln (residual sum of squares) + n ln(T),

Where T is the number of usable observations, and n is the number of parameters to be estimated.

4.0 Data Analysis

In order to illustrate volatility models, the monthly data on Nigerian crude oil prices from January 2015 to July 2020 are taken.

In order to estimate and forecast the volatility models of Nigerian crude oil prices four different GARCH (p,q) – type models will be used.

4.1 Graphical Representations of the Crude Oil Prices

The graph of the data is shown in figure 4.1

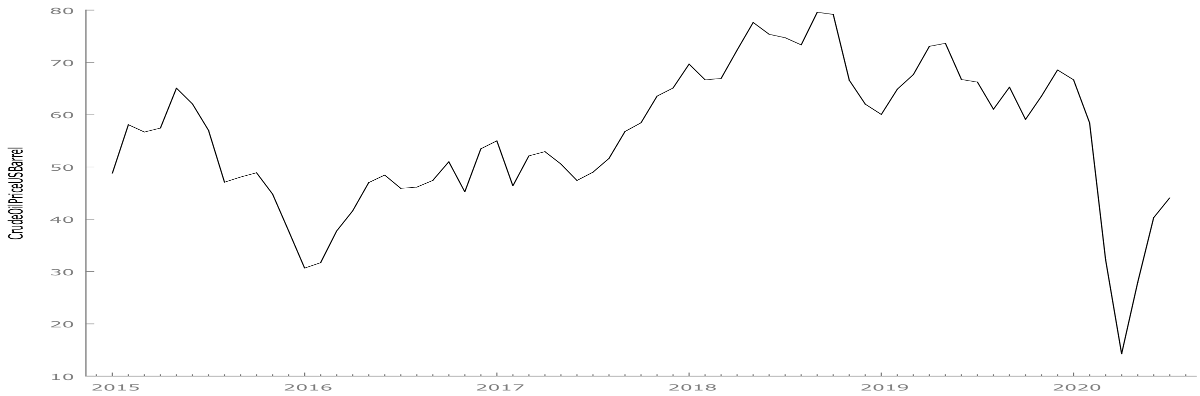

Figure 4.1 plot of the daily oil prices

It is easily seen that the series contains a trend component which should be removed before modeling. To remove the trend, we take the value of the first difference of logarithms of the levels which is the returns. Return series are preferred over price in analysis of financial time series because they have attractive statistical property like stationarity.

Figure 4.2 plot of the price return of crude oil prices

The plot of the returns in fig. 4.2 shows that some periods are riskier than others. Also, the risky times are scattered randomly. The amplitudes of the returns vary over time as large (small) changes in returns are followed by large (small) changes. This phenomenon is called **Volatility clustering** (Engle and Patton, 2001) and is one of the stylized facts of the financial time series. The plot also depicted as time went by, the shocks tend to persist, giving long memory process with high volatility half – lives in the returns. This phenomenon is called **volatility persistence** which is another stylized fact of the financial time series (Engle and Patton, 2001).

Since from the plot of the first difference, and it suggest that a time varying volatility and volatility clustering is quite evident in the data. From the plot, the first differenced series is stationary. However, to test statistically again ADF and KPSS unit test applied.

4.2 Unit Root Test for the Prices Returns

The ADF statistic tests the hypothesis of presence of unit root against the alternative of no unit root and the decision rule is to reject the null hypothesis when the value of test statistic is less than the critical value. The KPSS statistic tests the null hypothesis of stationarity against the alternative of non-stationarity and the decision rule is to accept the null hypothesis when the value of test statistic is less than the critical value. The results of the ADF and KPSS test are in Table 4.01.

Table 4.01 Unit root test for the price returns

Critical Values	ADF Test Statistics: -38.0111	KPSS Test Statistics: 0.05305
1%	-3.48	0.216
5%	-2.89	0.146
10%	-2.57	0.119

According to table 4.01 the ADF test statistics is greater than all the critical values in absolute value so the hypothesis of non – stationarity is rejected. Also is shown that the KPSS test statistics is less than the critical the hypothesis is accepted.

That means the differenced series is stationery which is denoted as *I(0)*

4.3 Testing for Arch Effects

Now, that we have shown that the series are stationery, our focus here is to test for the ARCH effects in the residuals of the returns and to establish if the ARCH effects are due to structural breaks or not.

Table 4.02: Results of ARCH – LM test, assuming there are no structural breaks

Test Statistic	P – Value (chi^2)
60.5109	0.0000

Table 4.03: Results of ARCH LM test for structural breaks for priced returns

	Sub period 1 2015 – 2017	Sub period 2 2018 – 2020
Test statistics	0.4326	51.2345
P – Value (chi^2)	0.97201	0.0000

The results in Table 4.02 – 4.03 show that whether or not we assume structural breaks in returns, there is evidence of ARCH effects for all the returns across the groups and they can be modeled as conditional heteroscedastic model. The ARCH effects are not due to structural breaks as there are ARCH effects across the sub groups.

4.5 Jacque Bera Test for Normality.

To achieve the first objective of the research, we examine the characteristics of unconditional distribution of price returns. This will enable us to explore and explain some stylized facts embedded in the financial returns. Jaque Bera normality test is used to demonstrate this and the results are given in Table 4.05

Table 4.05 Jaque Bera Test for Normality

Mean	0.01957
Median	0.02000
Variance	1.5109
Maximum	10.630
Minimum	-11.840
Std. Dev.	1.2292
Skewness	-0.5966
Kurtosis	13.9441
Jacque Berra Test	15801.2
P value	0.0000

The results in Table 4.05 show that all small positive average returns of about one – thousandth of a percent per day will be recorded for price return and the daily variance of 1.5109 and annualized volatility of 24.46%. The skewness coefficient indicates that the returns distribution is substantially negatively skewed; a common feature of equity returns. Finally, the kurtosis coefficient, which is a measure of the thickness of the tails of the distribution, is very high. A Gaussian distribution has kurtosis of 3, exhibit high kurtosis. This is one of the stylized facts known in the early days of volatility modeling. And also, from Jacque – Berra test, the hypothesis of normality is strongly rejected.

4.6 A Model Identification

The particular optimization routine of the log – likelihood function shows among GARCH(p,q) classes for p ϵ [1,2] and q ϵ [1,2], the optimal model for the returns is GARCH (1,1) model. The optimal model for the returns is GARCH (1,1) model. The results for the estimate of the log – liklihood are presenteted in table 4.06 for further information see Berdict *et al* (1974).

Table 4.06 the results of the estimate of the likelihood function for GARCH models

Model	Log likelihood of oil price returns
GARCH (1,1)	-3.1415×10^3
GARCH(1,2)	-3.1410×10^3
GARCH (2,1)	-3.1402×10^3
GARCH(2,2)	-3.1401×10^3

The results in table 4.06 suggest that as we increase the other of the model, the optimization process improves and this implies that GARCH (p,q) with sufficient number of terms can capture every volatility. To avoid such problem of number of terms sufficiency and parsimony, we resolve to implement GARCH (1, 1) as its optimality among GARCH (p, q) models.

4.7 Estimation of the Model

We estimate the parameters of GARCH (1,1) as the optimal model. We first assume Gaussian errors distribution for the innovations ε_t and later extend to non – Gaussian errors. The results of the estimated parameters are given in table 4.9 below:

Table 4.07 results of the estimated GARCH (1,1) parameters

	Coefficient	Standard error
Constant	0.0279	0.0127
α_0	0.00157	0.0006
α_1	0.0430	0.0053
β_1	0.9554	0.0052
Akaike Criterion	8.781	
Scwarzt Criterion	8.788	

4.8 Model Checking

Before the interpretation and the use of the model, we first look at some test and plots to whether the volatility models have adequately captured all of the persistence in the variance of returns. If the model is adequate, then the standardized squared residuals should be serially uncorrelated.

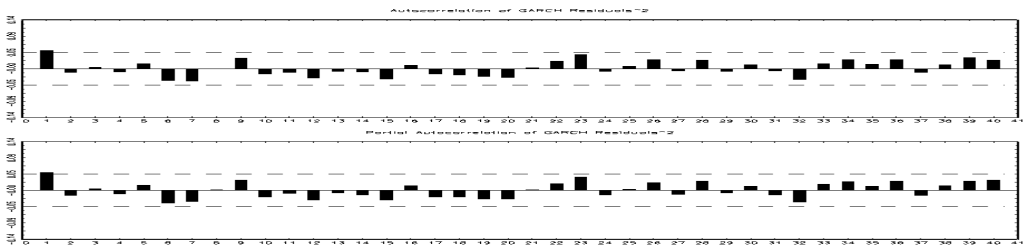

Fig.4.6 GARCH (1,1) residual of oil returns

Figure 4.6 show that there is no serial correlation observed in the residuals of the returns.

Table 4.09 results for no remaining ARCH effects test in residual

F – Test	P – value
1.1089	0.2925

In the table 4.09 above, the null hypothesis that there is no ARCH effects remaining at every lag is accepted since all the p – values are greater than 0.05. Therefore, the models are adequate.

Table 4.10 results of ARCH LM test for GARCH (1,1) residuals

Lag	Test Statistics	P – value
1	4.8829	0.0271
2	5.2098	0.0739
3	5.2976	0.1513
4	5.4315	0.2458

The null hypothesis of no ARCH effects should be accepted as in table 4.10 and the conclusion is that the models are adequate.

4.9 Forecast Performance

The models were also evaluated in terms of their forecasting ability of future returns. In table 4.11 below, the results of the forecast performance are shown. The model that exhibits the lowest values of the error measurements is to be the best one. The result shows that the GARCH (1, 1) model outperformed all the other models.

Table 4.11 forecast performance of estimated models

	GARCH(1,1)	GARCH(1,2)
Standard error	0.010088	0.010263

4.10 Forecast Evaluation

After a good volatility model have the ability to forecast (validation) and capture the commonly stylized facts. The ability to do so will further testify the validity of such models. Here, are the ten months' intervals forecast for the models. Starting from August 2020 t0 May 2021

Table 4.12 the results of the forecast performance validation

FORECAST RANGE: [2020 M8, 2021 M5], T = 10

TIME	LOWER CI	FORECAST	UPPER CI	STD. ERR
2020 M8	32.6771	44.8412	57.0054	6.2063
2020 M9	23.9794	44.7088	65.4381	10.5764
2020 M10	18.4727	44.7091	70.9456	13.3862
2020 M11	13.8600	44.6893	75.5186	15.7295
2020 M12	9.8585	44.6725	79.4865	17.7626
2021 M1	6.2669	44.6553	83.0436	19.5862
2021 M2	2.9812	44.6381	86.2950	21.2539
2021 M3	-0.0662	44.6209	89.3080	22.8000
2021 M4	-2.9207	44.6037	92.1282	24.2476
2021 M5	-5.6152	44.5866	94.7883	25.6136

4.11 Conclusions

The goal of this research work has been to characterize a good volatility model by its ability to forecast and capture the commonly held stylized facts about conditional volatility. The stylized facts include such things as the persistence in volatility, its mean reverting behavior, the asymmetric impact of negative versus positive return innovations and the possibility that exogenous or pre-determined variables may have a significant influence on volatility.

We used a monthly data form January, 2015 to July, 2020 on the prices on Nigeria crude oil to illustrate the ability of models from the GARCH family to capture these characteristics.

The GARCH (1,1) model is able to model and forecast better than other competing models. The forecast results show a slight upward movement in the crude oil prices.

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